MA 1323 Practice Exam 1 Solutions

1.) use $60' = 1^\circ$ and $3600'' = 1^\circ$ and $60'' = 1'$. for (a) $\frac{30'}{1}\left(\frac{1}{60'}\right)^\circ = .5^\circ$ so $35^\circ 30' = 35.5^\circ$

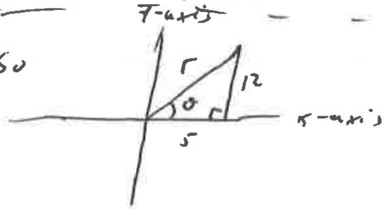
(b) now $(.75)(60') = 45'$ so $46.75^\circ = 46^\circ 45'$

2.) like in 1.) use same conversion. For (a) $\frac{54'}{1'}\left(\frac{1}{60}\right)^\circ = .9^\circ$ and $\frac{36''}{1''}\left(\frac{1}{3600}\right)^\circ = .01^\circ$

so $20^\circ 54' 36'' = 20^\circ + .9^\circ + .01^\circ = 20.91^\circ$

(b) now $(.4296)(60') = 25.776'$ and $(.776)(60'') = 46.56'' \approx 47''$ so $31.4296^\circ \approx 31^\circ 25' 47''$

3.) have terminal pt $(5,12)$ so



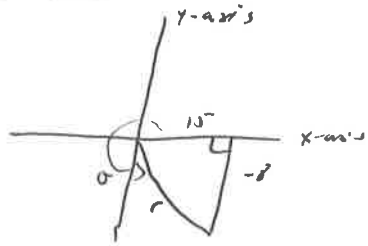
and $r = \sqrt{5^2 + 12^2} = \sqrt{25+144} = \sqrt{169} = 13$

so $\sin\theta = \frac{12}{13}$ $\quad \csc\theta = \frac{13}{12}$

$\cos\theta = \frac{5}{13}$ $\quad \sec\theta = \frac{13}{5}$

$\tan\theta = \frac{12}{5}$ $\quad \cot\theta = \frac{5}{12}$

4.) have $(15,-8)$ as terminal pt. so



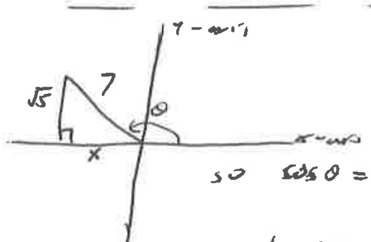
so $r = \sqrt{(-8)^2 + 15^2} = \sqrt{64+225} = \sqrt{289} = 17$

so $\sin\theta = \frac{-8}{17}$ $\quad \csc\theta = \frac{17}{-8}$

$\cos\theta = \frac{15}{17}$ $\quad \sec\theta = \frac{17}{15}$

$\tan\theta = \frac{-8}{15}$ $\quad \cot\theta = \frac{-15}{8}$

5.) have $\sin\theta = \frac{\sqrt{5}}{7}$, θ in QII: so



know $x^2 + (\sqrt{5})^2 = 7^2 \Rightarrow x^2 + 5 = 49$

$\Rightarrow x^2 = 44 \Rightarrow x = \pm\sqrt{44} = \pm 2\sqrt{11}$

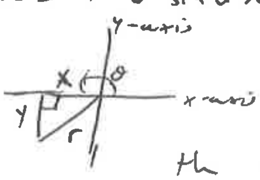
but $x < 0$ in QII so $x = -2\sqrt{11}$

so $\cos\theta = \frac{-2\sqrt{11}}{7}$ $\quad \csc\theta = \frac{7}{\sqrt{5}}$ $\quad \cot\theta = \frac{-2\sqrt{11}}{\sqrt{5}}$

$\tan\theta = \frac{-\sqrt{5}}{2\sqrt{11}}$ $\quad \sec\theta = \frac{-7}{2\sqrt{11}}$

6.) have $\sec\theta = -4$ and $\sin\theta > 0$ so $\sin\theta = \frac{y}{r}$ and $r > 0$ so $y < 0$ but $\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$ and $\sec\theta < 0 \Rightarrow x < 0$

in QIII



write $-4 = \frac{4}{-1}$ so $r = 4$, $x = -1$ then $y^2 + (-1)^2 = 4^2 \Rightarrow y^2 + 1 = 16$

$\Rightarrow y^2 = 15$ so $y = \pm\sqrt{15}$ but in QIII so $y = -\sqrt{15}$

then $\sin\theta = \frac{-\sqrt{15}}{4}$ $\quad \csc\theta = \frac{-4}{\sqrt{15}}$

$\cos\theta = \frac{-1}{4}$ $\quad \cot\theta = \frac{1}{\sqrt{15}}$

$\tan\theta = \sqrt{15}$

7.) we have the picture

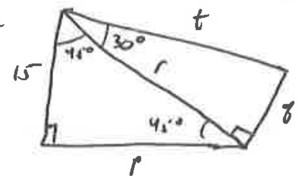
so $\sin 30° = \frac{y}{9} \Rightarrow y = 9\sin 30° = 9\cdot\frac{1}{2} = \frac{9}{2}$

and $\frac{x}{9} = \cos 30° \Rightarrow x = 9\cos 30° = 9\frac{\sqrt{3}}{2} = \frac{9\sqrt{3}}{2}$

Similarly $\frac{y}{z} = \tan 60° \Rightarrow z = \frac{\frac{9}{2}}{\tan 60°} = \frac{\frac{9}{2}}{\sqrt{3}} = \frac{9}{2\sqrt{3}}$

and finally $\frac{y}{w} = \sin 60° \Rightarrow w = \frac{y}{\sin 60°} = \frac{\frac{9}{2}}{\frac{\sqrt{3}}{2}} = \frac{9}{\sqrt{3}}$

8.) have the picture



since bottom Δ is isosceles $\Rightarrow p = 15$ so $r^2 = 15^2 + 15^2 = 450$
$\Rightarrow r = 15\sqrt{2}$ then $\cos 30° = \frac{15\sqrt{2}}{t} \Rightarrow \frac{\sqrt{3}}{2} t = 15\sqrt{2} \Rightarrow t = \frac{30\sqrt{2}}{\sqrt{3}}$

similarly $\tan 30° = \frac{q}{r} = \frac{q}{15\sqrt{2}}$ and $\tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{1}{\sqrt{3}} = \frac{q}{15\sqrt{2}}$

$\Rightarrow q = 15\sqrt{6}$

9.) have equation: $\tan(\alpha) = \cot(\alpha + 10°)$. Recall $\tan\theta = \cot(90° - \theta)$

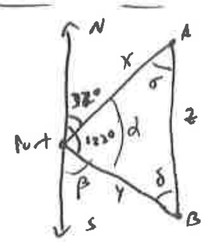
so $\cot(90° - \alpha) = \cot(\alpha + 10°)$ true iff $90° - \alpha = \alpha + 10° \Rightarrow 80° = 2\alpha$

$\Rightarrow \alpha = 40°$ in $[0, 360°)$ also have $\alpha = 40° + 180° = 220°$.

10.) have $\cos\theta = \sin(2\theta - 30°)$ recall $\cos\theta = \sin(90° - \theta) \Rightarrow \sin(90° - \theta) = \sin(2\theta - 30°)$

true iff $90° - \theta = 2\theta - 30° \Rightarrow 3\theta = 120° \Rightarrow \theta = 40°$ in $[0, 360°)$ also have $\theta = 40° + 180° = 220°$

11.) have following picture



x-dist of 1st ship, y-dist of 2nd ship, z dist between ships

but $x = (16\ \text{mi/hr})(2.5\ \text{hr}) = 40$ miles, $y = (24\ \text{mi/hr})(2.5\ \text{hr}) = 60$ miles

and $\beta + 122° = 180° \Rightarrow \beta = 58°$ so $\alpha + 32° + 58° = 180° \Rightarrow \alpha = 90°$

so $\delta = 32°$ by alternate interior so $\frac{x}{z} = \cos 32°$

and $\cos 32° \approx .848$ so $z = \frac{40}{.848} = 47.17$ miles

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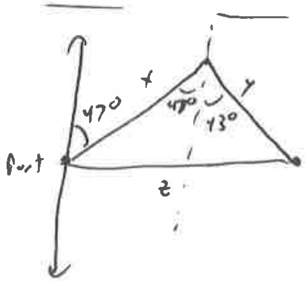
12) have following picture similar to 11.)



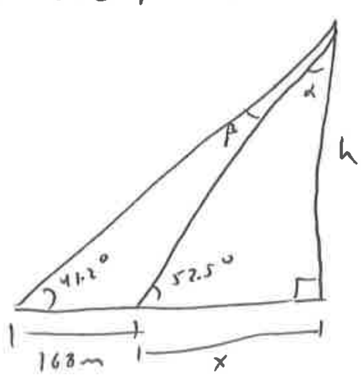
see $47° + 43° = 90°$ form right Δ. $x = 22(3.5) = 77$ miles

$y = (4)(22) = 88$ miles

but $z^2 = x^2 + y^2 \Rightarrow z^2 = 88^2 + 77^2 = 7744 + 5929 = 13673$

so $z = \sqrt{13673} \approx 116.9$ miles

13.) have picture



since $\alpha + 52.5° + 90° = 180° \Rightarrow \alpha = 39.5°$ and $\beta + \alpha + 41.2° + 90° = 180°$

$\Rightarrow \beta = 9.3°$ then $\tan\alpha = \frac{x}{h}$ and $\tan(\alpha+\beta) = \frac{x+168}{h} \Rightarrow \frac{\tan(\alpha+\beta)}{\tan\alpha} = \frac{x+168}{x}$

$\tan\alpha \approx .824$, $\tan(\alpha+\beta) = \tan(48.8°) \approx 1.142$

$\Rightarrow \frac{1.142}{.824} = \frac{x+168}{x} \Rightarrow 1.38 = \frac{x+168}{x} \Rightarrow 1.38x = x + 168 \Rightarrow .38x = 168$

$\Rightarrow x = 442.11$ so $x + 168 = 610.11$ and finally $1.142 = \frac{610.11}{h}$

$\Rightarrow h = 534.25$ m.