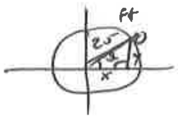
MA 1323 Practice Exam 2 Solutions

1.) wat $\cos(4\theta+\pi)$ if $\theta = \frac{\pi}{4}$ so $\Rightarrow$ $\cos(4(\frac{\pi}{4})+\pi) = \cos(\pi+\pi) = \cos(2\pi) = 1$

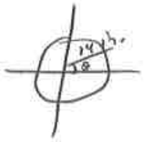
2.) wat $\sin(5\theta+\pi)$ if $\theta = \frac{\pi}{2}$. so $\sin(\frac{5\pi}{2}+\pi) = \sin(\frac{5\pi}{2}+\frac{2\pi}{2}) = \sin(\frac{7\pi}{2}) = -1$

3.)



$\theta = \frac{5\pi}{6}$ then $y = 25\sin(\frac{5\pi}{6}) = 25(\frac{1}{2}) = \frac{25}{2}$ ft.. so $\omega = \frac{\theta}{t} = \frac{\frac{\pi}{3}}{30} = \frac{\pi}{90}$ rads/sec

4.)



so then $r = 14$ in. & $v = 55$ mph & $v = r\omega$ so & $1\text{ mi} = 63{,}360$ inches

so $r = \left(\frac{14\text{ in}}{63{,}360\text{ in}}\right)$ mi & $\omega = \left(\frac{55}{\frac{14}{63{,}360}}\right) = \frac{55}{14}(63{,}360) \approx 248{,}914.28$ rotations

now if $r = 16$ in. then $v = \left(\frac{16}{63{,}360}\right)\left(\frac{55}{14}\right)(63{,}360) \approx 62.85$ mph & $d = vt$ w/ $t=1$

$\Rightarrow$ distance ≈ 62.85 miles, yes he's speeding

5.) $f(x) = 3+4\cos(3x+\pi)$, then $0 \le 3x+\pi \le 2\pi \Rightarrow$ on $[-\frac{\pi}{3}, \frac{\pi}{3}]$ then $p = \frac{\pi}{3}-(-\frac{\pi}{3}) = \frac{2\pi}{3}$

the ampl. $= |4| = 4$, when $x=0$ $\cos(\pi) = -1$ so $f(0) = 3-4 = -1$ so y-inct. $= -1$

fully write $f(x) = 3+4\cos(3(x+\frac{\pi}{3}))$ see phase shift is $|-\frac{\pi}{3}| = \frac{\pi}{3}$

6.) $f(x) = -3+\sin(x+\frac{\pi}{2})$ then $0 \le x+\frac{\pi}{2} \le 2\pi \Rightarrow$ on int. $[-\frac{\pi}{2}, \frac{3\pi}{2}]$ & $p = \frac{3\pi}{2}-(-\frac{\pi}{2}) = 2\pi$

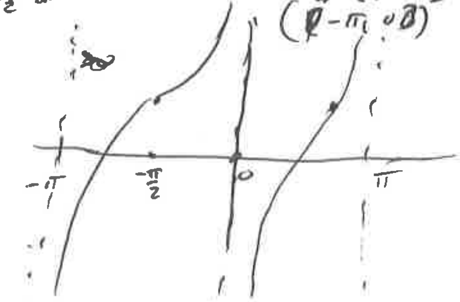
the ampl. $= |1| = 1$ when $x=0$ $f(0) = -3+\sin(\frac{\pi}{2}) = -3+1 = -2$ so y-inct. $= -2$

the phase shift is $|-\frac{\pi}{2}| = \frac{\pi}{2}$

7.) $f(x) = 1-\tan(x+\frac{\pi}{2})$ then $-\frac{\pi}{2} \le x+\frac{\pi}{2} \le \frac{\pi}{2} \Rightarrow$ on int. $(-\pi, 0)$ w/ $p = \pi$

& ampl. $= |-1| = 1$ $f(-\frac{\pi}{2}) = 1$

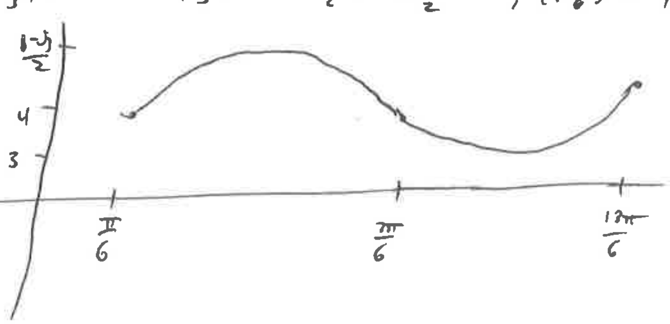
so



8.) $f(x) = 4-\sin(2x-\frac{\pi}{3})$ then $0 \le 2x-\frac{\pi}{3} \le 2\pi \Rightarrow \frac{\pi}{3} \le 2x \le \frac{7\pi}{3} \Rightarrow \frac{\pi}{6} \le x \le \frac{7\pi}{6}$ on int. $[\frac{\pi}{6}, \frac{7\pi}{6}]$ w/ $p = \pi$

& ampl. $= |-1| = 1$ then $f(\frac{\pi}{3}) = 4-\sin(\frac{\pi}{3}) = 4-\frac{\sqrt{3}}{2} = \frac{8-\sqrt{3}}{2}$, $f(\frac{\pi}{6}) = 4$, $f(\frac{7\pi}{6}) = 4$

so



9.) Want to show $\sin\theta + \cos\theta = \dfrac{\sin\theta}{1-\cot\theta} + \dfrac{\cos\theta}{1-\tan\theta}$ using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Consider RHS $= \dfrac{\sin\theta}{1-\cot\theta} + \dfrac{\cos\theta}{1-\tan\theta} = \dfrac{(1-\tan\theta)\sin\theta + (1-\cot\theta)\cos\theta}{(1-\tan\theta)(1-\cot\theta)} = \dfrac{\sin^2\theta\cos\theta - \sin^3\theta + \cos^2\theta\sin\theta - \cos^3\theta}{2\sin\theta\cos\theta - \sin^2\theta - \cos^2\theta}$

$= \dfrac{(1-\cos^2\theta)\cos\theta + (1-\sin^2\theta)\sin\theta - (\sin^3\theta + \cos^3\theta)}{2\sin\theta\cos\theta - 1} = \dfrac{\sin\theta+\cos\theta - 2(\cos^3\theta+\sin^3\theta)}{2\sin\theta\cos\theta - 1}$, using $x^3+y^3=(x^2-xy+y^2)(x+y)$

$= \dfrac{\sin\theta+\cos\theta - 2(\sin\theta+\cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)}{2\sin\theta\cos\theta - 1} = \dfrac{(\sin\theta+\cos\theta)(2\sin\theta\cos\theta - 1)}{2\sin\theta\cos\theta-1} = \sin\theta+\cos\theta = LHS.$

10.) Want to show $\dfrac{\sin(s+t)}{\cos s\cos t} = \tan s + \tan t$

Consider LHS $= \dfrac{\sin(s+t)}{\cos s\cos t} = \dfrac{\sin s\cos t + \sin t\cos s}{\cos s\cos t} = \dfrac{\sin s\cos t}{\cos s\cos t} + \dfrac{\sin t\cos s}{\cos s\cos t} = \dfrac{\sin s}{\cos s} + \dfrac{\sin t}{\cos t} = \tan s + \tan t = RHS.$

11.) Want to show $\sin(x+y) + \sin(x-y) = 2\sin x\cos y$

Consider LHS $= \sin(x+y) + \sin(x-y) = \sin x\cos y + \sin y\cos x + \sin x\cos y - \sin y\cos x = 2\sin x\cos y = RHS$

12.) Want to show $\dfrac{\tan\varphi}{1+\cos\varphi} + \dfrac{\sin\varphi}{1-\cos\varphi} = \cot\varphi + \sec\varphi\csc\varphi$

Consider LHS $= \dfrac{\tan\varphi}{1+\cos\varphi} + \dfrac{\sin\varphi}{1-\cos\varphi} = \dfrac{\tan\varphi(1-\cos\varphi) + \sin\varphi(1+\cos\varphi)}{(1+\cos\varphi)(1-\cos\varphi)} = \dfrac{\tan\varphi - \sin\varphi + \sin\varphi + \sin\varphi\cos\varphi}{1-\cos^2\varphi}$

$= \dfrac{\tan\varphi + \sin\varphi\cos\varphi}{\sin^2\varphi} = \dfrac{\tan\varphi}{\sin^2\varphi} + \dfrac{\cos\varphi}{\sin\varphi} = \dfrac{\sin\varphi}{\cos\varphi}\cdot\dfrac{1}{\sin^2\varphi} + \cot\varphi = \sec\varphi\csc\varphi + \cot\varphi = RHS.$

13.) Want to show $\cos(2x) = \dfrac{\cot^2 x - 1}{\cot^2 x + 1}$

Consider RHS $= \dfrac{\cot^2 x - 1}{\cot^2 x + 1} = \dfrac{\frac{\cos^2 x}{\sin^2 x} - 1}{\frac{\cos^2 x}{\sin^2 x} + 1} = \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} = \dfrac{\cos(2x)}{1} = \cos(2x) = LHS$