

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Verify the following identity:

$$\tan x + \cot x = 2 \csc(2x)$$

2. Verify the following identity:

$$\frac{2 \cos(2\theta)}{\sin(2\theta)} = \cot \theta - \tan \theta$$

3. Evaluate the following expression without a calculator (The answer must be an exact number):

$$\sin \left(\tan^{-1}(-3) + \sin^{-1} \left(\frac{1}{2} \right) \right)$$

4. Evaluate the following expression without a calculator (The answer must be an exact number):

$$\cos \left(\tan^{-1} \left(\frac{5}{12} \right) - \tan^{-1} \left(\frac{3}{4} \right) \right)$$

5. Solve $\tan x + \sqrt{3} = \sec x$ over the interval $[0, 2\pi)$.

6. Solve $2 \cos^2 x - \cos x = 1$ over the interval $[0, 2\pi)$.

7. Find all solutions of the equation $-2 \sin^2 x = 3 \sin x + 1$.

8. Find all solutions of the equation $\tan x(\tan x - 2) = 5$.

9. The bearing of a lighthouse from a ship was found to be N 37° E. After the ship sailed 2.5 miles due south, the new bearing was N 25° E. Find the distance between the ship and the lighthouse at each location.

10. Standing on one bank of a river flowing north, Louis notices a tree on the opposite bank at a bearing of 115.45° . Gertrude is on the same bank as Louis, but 428.3 m away. She notices that the bearing of the tree is 45.47° . The two banks are parallel. What is the distance across the river?

11. Find the exact measure of the angle θ in the following triangle.

12. Find the length of the remaining side in the following triangle.

13. The distance from LA to NY is 2541 miles, from NY to Montreal is 331 miles, and from Montreal to LA is 2427 miles. What is the area of the triangular region having these three cities as vertices? (Ignore the Earth's curvature.)