MA 1323 Practice Final Exam Solutions

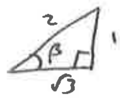
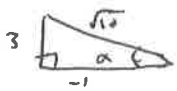
1.) want to show $\tan x + \cot x = 2\csc(2x)$

consider LHS $= \tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = \frac{2}{2\sin x \cos x} = \frac{2}{\sin(2x)} = 2\csc(2x) =$ RHS

2.) want to show $\frac{2\cos(2\theta)}{\sin(2\theta)} = \cot\theta - \tan\theta$

consider LHS $= \frac{2\cos(2\theta)}{\sin(2\theta)} = \frac{2(\cos^2\theta - \sin^2\theta)}{2\sin\theta\cos\theta} = \frac{\cos^2\theta}{\sin\theta\cos\theta} - \frac{\sin^2\theta}{\sin\theta\cos\theta} = \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta - \tan\theta =$ RHS

3.) consider $\sin\left(\tan^{-1}(-3) + \sin^{-1}\left(\tfrac12\right)\right)$ let $\alpha = \tan^{-1}(-3)$ and $\beta = \sin^{-1}\left(\tfrac12\right)$ $\Rightarrow$ $\tan\alpha = -3$ and $\sin\beta = \frac12$

so  and  since $\sec^2\alpha = 1 + \tan^2\alpha = 1 + 9 = 10$

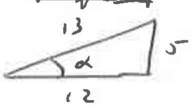
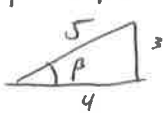
$\Rightarrow$ so $\sec\alpha = \sqrt{10}$ $\Rightarrow$ $\cos\alpha = -\frac{1}{\sqrt{10}}$

then $\sin(\alpha+\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha = \frac{3}{\sqrt{10}}\left(\frac{\sqrt3}{2}\right) - \frac12\left(\frac{1}{\sqrt{10}}\right) = \frac{3\sqrt3}{2\sqrt{10}} - \frac{1}{2\sqrt{10}} = \frac{3\sqrt3 - 1}{2\sqrt{10}}$

4.) consider $\cos\left(\tan^{-1}\left(\frac{5}{12}\right) - \tan^{-1}\left(\frac34\right)\right)$ let $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$ and $\beta = \tan^{-1}\left(\frac34\right)$ $\Rightarrow$ $\tan\alpha = \frac{5}{12}$, $\tan\beta = \frac34$

but $\sec^2\alpha = 1 + \tan^2\alpha = 1 + \frac{25}{144} = \frac{169}{144}$ $\Rightarrow$ $\sec\alpha = \pm\frac{13}{12}$ and similarly $\sec^2\beta = 1 + \tan^2\beta = 1 + \frac{9}{16}$ $\Rightarrow$ $\sec\beta = \pm\frac54$

have  so $\sin^2\beta + \cos^2\beta = 1$ $\Rightarrow$ $\sin^2\beta = 1 - \frac{16}{25} = \frac{9}{25}$ $\Rightarrow$ $\sin\beta = \pm\frac35$ and similarly $\sin\alpha = \pm\frac{5}{13}$ since $\tan\alpha, \tan\beta > 0$

$\Rightarrow$  ,  so $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{5}{12}\left(\frac45\right) + \frac{5}{13}\left(\frac35\right) = \frac13 + \frac{3}{13} = \frac{48}{39}$

5.) have $\tan x + \sqrt3 = \sec x$ on $[0, 2\pi)$

then $(\tan x + \sqrt3)^2 = \sec^2 x$ $\Rightarrow$ $\tan^2 x + 2\sqrt3\tan x + 3 = 1 + \tan^2 x$ $\Rightarrow$ $2\sqrt3\tan x + 3 = 1$ $\Rightarrow$ $2\sqrt3\tan x = -2$

$\Rightarrow$ $\tan x = -\frac{1}{\sqrt3}$, know $\tan x' = \frac{1}{\sqrt3}$ in QI $\Rightarrow$ $x' = \frac{\pi}{6}$ so with $\tan x < 0$ in QII or QIV

$\Rightarrow$ $x = \frac{5\pi}{6}$ or $\frac{11\pi}{6}$ need to check $\tan\left(\frac{11\pi}{6}\right) + \sqrt3 = -\frac{2\sqrt3}{3}$ but $\sec\left(\frac{11\pi}{6}\right) = \frac{2\sqrt3}{3}$ but $\tan\left(\frac{5\pi}{6}\right) + \sqrt3 = \frac{2\sqrt3}{3}$ and $\sec\left(\frac{5\pi}{6}\right) = -\frac{2\sqrt3}{3}$

only include $x = \frac{11\pi}{6}$

6.) have $2\cos^2 x - \cos x = 1$ on $[0, 2\pi)$

let $y = \cos x$ $\Rightarrow$ $2y^2 - y = 1$ $\Rightarrow$ $2y^2 - y - 1 = 0$ $\Rightarrow$ $(2y+1)(y-1) = 0$ $\Rightarrow$ $2\cos x + 1 = 0$ or $\cos x - 1 = 0$

$\Rightarrow$ $\cos x = -\frac12$ or $\cos x = 1$. $\cos x = 1$ $\Rightarrow$ $x = 0$, and $\cos x = -\frac12$ $\Rightarrow$ $x = \frac{2\pi}{3}, \frac{5\pi}{3}$.

7.) have $-2\sin^2 x = 3\sin x + 1$, let $y = \sin x$ $\Rightarrow$ $-2y^2 = 3y + 1$ $\Rightarrow$ $2y^2 + 3y + 1 = 0$ $\Rightarrow$ $(2y+1)(y+1) = 0$

$\Rightarrow$ $2\sin x + 1 = 0$ or $\sin x + 1 = 0$ $\Rightarrow$ $\sin x = -\frac12$ or $\sin x = -1$. 1st $\sin x = -1$ $\Rightarrow$ $x = \frac{3\pi}{2}$ in $[0, 2\pi)$

but for all values $x = \frac{3\pi}{2} + 2n\pi$, $n \in \mathbb{Z}$. 2nd $\sin x = -\frac12$ $\Rightarrow$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$ in $[0, 2\pi)$

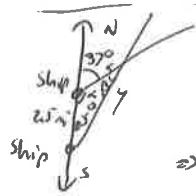
for all values need $x = \frac{7\pi}{6} + 2n\pi$ and $\frac{11\pi}{6} + 2n\pi$ for $n \in \mathbb{Z}$

8.) have $\tan x(\tan x - 2) = 5 \Rightarrow \tan^2 x - 2\tan x - 5 = 0$, let $y = \tan x \Rightarrow y^2 - 2y - 5 = 0$

so $y = \frac{2 \pm \sqrt{4 - 4(-5)}}{2} = \frac{2 \pm \sqrt{24}}{2} = \frac{2 \pm 2\sqrt{6}}{2} = 1 \pm \sqrt{6}$ so $\tan x = 1+\sqrt{6}$ or $\tan x = 1-\sqrt{6}$

for 1st for $\tan x = 1+\sqrt{6} \Rightarrow x \approx 1.3$ want all solns so $x = 1.3 + 2n\pi$ and for $\tan x = 1-\sqrt{6} \Rightarrow x \approx -0.967$ want all solns so $x = -0.967 + 2n\pi$

9.) have the picture



want x, y. then $\alpha + 37° = 180° \Rightarrow \alpha = 143°$ so $\alpha + \beta + 25° = 180°$

$\Rightarrow \beta + 168° = 180° \Rightarrow \beta = 12°$ so $\frac{\sin 12°}{2.5} = \frac{\sin 143°}{y}$ by law of sines

$\Rightarrow y = \left(\frac{\sin 143°}{\sin 12°}\right)(2.5) \approx$ 2.025 miles similarly $\frac{\sin 12°}{2.5} = \frac{\sin 25°}{x}$

$\Rightarrow x = \left(\frac{\sin 25°}{\sin 12°}\right)(2.5) \approx 1.58$ miles

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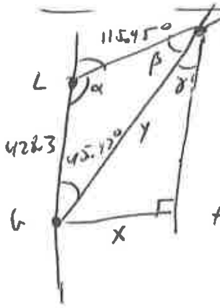
10.) have the picture



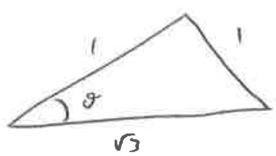
know $115.45° + \alpha = 180° \Rightarrow \alpha = 64.55°$ and $\alpha + \beta + 45.47° = 180°$

$\Rightarrow \beta = 69.68°$ and $\gamma = 45.47°$ alt. int. angles

so $\frac{\sin 69.68°}{428.3} = \frac{\sin 64.55°}{y}$ by law of sines $\Rightarrow y = \left(\frac{\sin 64.55°}{\sin 69.68°}\right)(428.3) \approx 410.1$

then finally $x = 410.1 \sin 45.47° = 292.4$m

11.) have

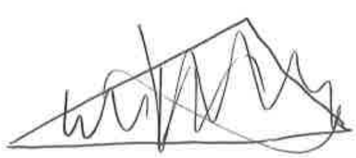


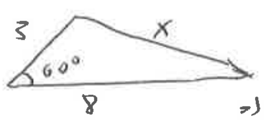
by law of cosines have $1^2 = 1^2 + (\sqrt{3})^2 - 2(\sqrt{3})(1)\cos\theta$

$\Rightarrow 1 = 1 + 3 - 2\sqrt{3}\cos\theta \Rightarrow -3 = -2\sqrt{3}\cos\theta \Rightarrow \cos\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

since θ is acute $\Rightarrow \theta = \frac{\pi}{6}$ or $30°$

12.) have





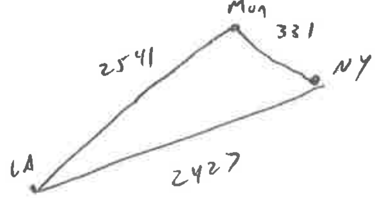
by law of cosines have

$x^2 = 3^2 + 8^2 - 2(8)(3)\cos(60°) = 9 + 64 - 48\left(\frac{1}{2}\right)$

$\Rightarrow x^2 = 73 - 24 = 49 \Rightarrow x = \pm\sqrt{49} = \pm 7$

but x is length so $x = 7$

13.) have picture



by Heron's formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ w/ $s = \frac{1}{2}(a+b+c)$

so $s = \frac{1}{2}(331 + 2541 + 2427) = \frac{5299}{2}$

then $s - a = \frac{5299}{2} - 2541 = \frac{217}{2}$, $s - b = \frac{5299}{2} - 2427 = \frac{445}{2}$

and $s - c = \frac{5299}{2} - 331 = \frac{4637}{2}$

so $A = \sqrt{\left(\frac{5299}{2}\right)\left(\frac{217}{2}\right)\left(\frac{445}{2}\right)\left(\frac{4637}{2}\right)} = \frac{1}{4}\sqrt{(5299)(217)(445)(4637)}$