MA 1613 Practice Exam 1 Solutions

1.) $m = \frac{6-2}{9--3} = \frac{4}{9+3} = \frac{4}{12} = \frac{1}{3}$, then $y-2 = \frac{1}{3}(x--3) \Rightarrow y-2 = \frac{1}{3}x+1 \Rightarrow y = \frac{1}{3}x+3$ ← slope intercept form

for standard: $\Rightarrow -\frac{1}{3}x + y = 3$ or $-x+3y=9$

2.) $m = \frac{10-3}{12-7} = \frac{7}{5}$, then $y-3 = \frac{7}{5}(x-7) \Rightarrow y-3 = \frac{7}{5}x - \frac{49}{5} \Rightarrow y = \frac{7}{5}x - \frac{34}{5}$ slope inter. form

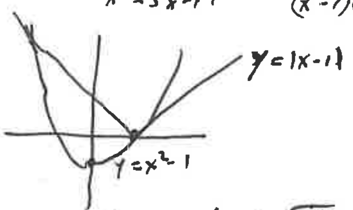
for standard: $\Rightarrow 5y = 7x-34$ or $7x-5y=34$.

3.) $f(x) = \frac{x^2+1}{\sqrt{x^2-3x+2}} = \frac{x^2+1}{\sqrt{(x-2)(x-1)}}$ need $x \neq 2$, $x \neq 1$ and $(x-2)(x-1) \geq 0$ so $x>2$ and $x>1$ or $x<2$ and $x<1$

so domain $= (-\infty, 1) \cup (2, \infty)$

4.) $f(x) = \frac{\sqrt{x-7}}{x^2-5x-14} = \frac{\sqrt{x-7}}{(x-7)(x+2)} = \frac{1}{\sqrt{x-7}(x+2)}$ need $x \neq -2$ and $x>7$ ∴ domain $= (7, \infty)$

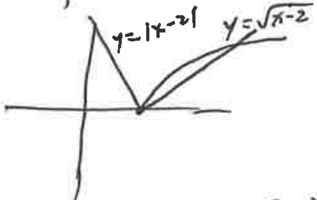
5.)



x	$y=\lvert x-1\rvert$
0	1
1	0
2	1
-1	2

x	$y=x^2-1$
0	-1
1	0
-1	0

6.)



x	$y=\lvert x-2\rvert$
0	2
2	0
1	1
3	1

x	$y=\sqrt{x-2}$
2	0
3	1
4	$\sqrt{2}$
6	2

7.) $\lim_{x\to1}\frac{x^4-1}{x^2-1} = \lim_{x\to1}\frac{(x^2-1)(x^2+1)}{x^2-1} = \lim_{x\to1}(x^2+1) = 1+1 = 2$

8.) $\lim_{x\to2}\frac{x^2-3x+2}{x^2-5x+6} = \lim_{x\to2}\frac{(x-1)(x-2)}{(x-2)(x-3)} = \lim_{x\to2}\frac{x-1}{x-3} = \frac{2-1}{2-3} = -1$

9.) check if $\lim_{x\to1} f(x) = f(1)$? $\lim_{x\to1}\frac{x^2-1}{x-1} = \lim_{x\to1}\frac{(x-1)(x+1)}{x-1} = \lim_{x\to1}(x+1) = 1+1 = 2 \neq -2) = f(1)$. so not cont. at $x=1$

10.) check if $\lim_{x\to1} f(x) = f(1)$? $\lim_{x\to1}\frac{\sqrt{x}-1}{x-1} = \lim_{x\to1}\frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x\to1}\frac{x-1}{(x-1)(\sqrt{x}+1)} = \lim_{x\to1}\frac{1}{\sqrt{x}+1} = \frac{1}{1+1} = \frac{1}{2} = f(1)$

so f is cont. at $x=1$.

11.) let $f(x) = \frac{|x|}{x}$ notice f is not defined at $x=0$ consider $\lim_{x\to0^+}\frac{|x|}{x} = \lim_{x\to0^+}\frac{x}{x} = \lim_{x\to0^+}1 = 1$. and $\lim_{x\to0^-}\frac{|x|}{x} = \lim_{x\to0^-}\frac{-x}{x} = \lim_{x\to0^-}-1 = -1$ so $\lim_{x\to0^-}f(x) \neq \lim_{x\to0^+}f(x)$ ∴ $\lim_{x\to0}f(x)$ DNE.

12.) $\lim_{x\to-2}\frac{x^3+8}{x^2-4} = \lim_{x\to-2}\frac{(x+2)(x^2-2x+4)}{(x+2)(x-2)} = \lim_{x\to-2}\frac{x^2-2x+4}{x-2} = \frac{4+4+4}{-4} = \frac{12}{-4} = -3$. ∴ limit exists.

13.) $\lim_{x\to0}\frac{\sqrt{4+x}-\sqrt{4-x}}{x} = \lim_{x\to0}\frac{\sqrt{4+x}-\sqrt{4-x}}{x}\cdot\frac{\sqrt{4+x}+\sqrt{4-x}}{\sqrt{4+x}+\sqrt{4-x}} = \lim_{x\to0}\frac{4+x-(4-x)}{x(\sqrt{4+x}+\sqrt{4-x})} = \lim_{x\to0}\frac{x}{x(\sqrt{4+x}+\sqrt{4-x})}$

$= \lim_{x\to0}\frac{1}{\sqrt{4+x}+\sqrt{4-x}} = \frac{1}{\sqrt{4}+\sqrt{4}} = \frac{1}{2+2} = \frac{1}{4}$.