MA 1613 : Practice Exam 2 Solutions

1.) $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \dfrac{(x+h)^2 - 2(x+h) + 3 - (x^2 - 2x + 3)}{h} = \lim_{h\to 0} \dfrac{x^2 + 2xh + h^2 - 2x - 2h + 3 - x^2 + 2x - 3}{h}$

$= \lim_{h\to 0} \dfrac{2xh - 2h + h^2}{h} = \lim_{h\to 0} (2x - 2 + h) = 2x - 2$

2.) $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \dfrac{2(x+h)^2 + (x+h) + 5 - (2x^2 + x + 5)}{h} = \lim_{h\to 0} \dfrac{2x^2 + 4xh + 2h^2 + x + h + 5 - 2x^2 - x - 5}{h}$

$= \lim_{h\to 0} \dfrac{4xh + h + 2h^2}{h} = \lim_{h\to 0} (4x + 1 + 2h) = 4x + 1$

3.) $f'(x) = 12x^{12-1} + 4(4)x^{4-1} - 6\cdot 2x^{2-1} + 3 = 12x^{11} + 16x^3 - 12x + 3$

4.) $f'(x) = 2\cdot 7x^{7-1} - 9\cdot 3x^{3-1} + 2\cdot 2x^{2-1} + 7 = 14x^6 - 27x^2 + 4x + 7$

5.) $f'(x) = \left(\dfrac{x^3}{x^2+5}\right)' + \left(\sqrt{x^3+8}\right)' = \dfrac{3x^2(x^2+5) - x^3(2x)}{(x^2+5)^2} + \left((x^3+8)^{\frac{1}{2}}\right)' = \dfrac{3x^4 + 15x^2 - 2x^4}{(x^2+5)^2}$

$+ \frac{1}{2}(x^3+8)^{-\frac{1}{2}}\cdot (x^3+8)' = \dfrac{x^4 + 15x^2}{(x^2+5)^2} + \frac{1}{2}\cdot 3x^2 \dfrac{1}{\sqrt{x^3+8}}$

6.) $f(x) = \left(\dfrac{x^3+3x+2}{x^2+2x-1}\right)^{\frac{1}{2}}$ so $f'(x) = \frac{1}{2}\left(\dfrac{x^3+3x+2}{x^2+2x-1}\right)^{-\frac{1}{2}}\cdot\left(\dfrac{x^3+3x+2}{x^2+2x-1}\right)' = \frac{1}{2}\left(\dfrac{x^3+3x+2}{x^2+2x-1}\right)^{-\frac{1}{2}} \dfrac{(3x^2+3)(x^2+2x-1) - (2x+2)(x^3+3x+2)}{(x^2+2x-1)^2}$

7.) $f'(x) = 4x + 1$ so $m = f'(0) = 4\cdot 0 + 1 = 1$ then $f(0) = 0^2 + 0 - 1 = -1$ the pt is $(0,-1)$

so $y - -1 = 4(x-0) \Rightarrow y + 1 = 4x \Rightarrow y = 4x - 1$

8.) $f'(x) = 21x^2 + 4x$ so $m = f'(1) = 21\cdot 1^2 + 4\cdot 1 = 21 + 4 = 25$., then $f(1) = 7 + 2 - 2 = 7$, the pt is $(1,7)$

so $y - 7 = 25(x-1) \Rightarrow y - 7 = 25x - 25 \Rightarrow y = 25x - 18$

9.) $f'(x) = 3x^2 - 4x^3 = x^2(3 - 4x)$ so $f'(x) = 0 \Rightarrow x^2 = 0$ or $3 - 4x = 0 \Rightarrow x = 0$ or $4x = 3 \Rightarrow x = \frac{3}{4}$

two crt #'s $x = 0$, $x = \frac{3}{4}$. next $f'(-1) = 3 + 4 = 7 > 0$ & $f'(\frac{1}{2}) = \frac{3}{4} - \frac{4}{8} = \frac{1}{4} > 0$ no change

in sign so $x = 0$ not max nor min. next $f'(\frac{1}{2}) = \frac{1}{4} > 0$ & $f'(1) = 3 - 4 = -1 < 0$

so f' goes from $+$ to $-$ $\Rightarrow f \nearrow$ to $\searrow$ at $x = \frac{3}{4}$ so relative max.

10.) $f'(x) = 12x^2 - 18x - 30 = 6(2x^2 - 3x - 5) = 6(2x - 5)(x+1)$ so $f'(x) = 0 \Rightarrow x + 1 = 0$ or $2x - 5 = 0$

$\Rightarrow x = -1$ or $x = \frac{5}{2}$. $f''(x) = 24x - 18$, then $f''(-1) = -24 - 18 < 0 \Rightarrow x = -1$ rel. max.

$f''(\frac{5}{2}) = (12)(\frac{25}{4}) - 18(\frac{5}{2}) - 30 = 75 - 30 - 30 = 15 > 0$ so $x = \frac{5}{2}$ rel. min.

11.) $f(x) = x^3 - 2x^2 - 4x + 3$, $f(0) = 3$. $f'(x) = 3x^2 - 4x - 4 = (3x+2)(x-2)$, $f''(x) = 6x - 4$

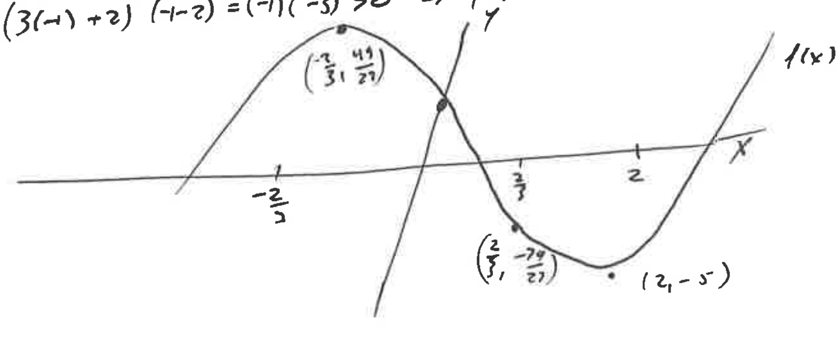
$f'(x) = 0 \therefore \Rightarrow 3x + 2 = 0$ or $x - 2 = 0 \Rightarrow x = 2$ or $x = \frac{-2}{3}$ are crt. #s. $f''(2) = 6(2) - 4 > 0$ so rel min

$f''(-\frac{2}{3}) = 6(-\frac{2}{3}) - 4 < 0 \Rightarrow$ rel. max. $f''(x) = 0 \Rightarrow 6x - 4 = 0 \Rightarrow x = \frac{4}{6} = \frac{2}{3}$ ← possible inflec. pt.

$f''(x) > 0$ for $x > \frac{2}{3}$ & $f''(x) < 0$ for $x < \frac{2}{3}$ so f concave up on $(\frac{2}{3}, \infty)$ concave down on $(-\infty, \frac{2}{3})$

$f'(0) = -4 < 0$ so $f \searrow$ on $(-\frac{2}{3}, 2)$, $f'(3) = (3(3)+2)(3-2) > 0 \Rightarrow f \nearrow$ on $(2, \infty)$,

$f'(-1) = (3(-1)+2)(-1-2) = (-1)(-3) > 0 \Rightarrow f \nearrow$ on $(-\infty, -\frac{2}{3})$. $f(2) = -5$, $f(-\frac{2}{3}) = \frac{49}{27}$, $f(\frac{2}{3}) = \frac{-29}{27}$



12) $f(x) = x^3 - 3x^2 - 144x - 140$, $f(0) = -140$, $f'(x) = 3x^2 - 6x - 144 = 3(x-8)(x+6)$. $f''(x) = 6x - 6 = 6(x-1)$

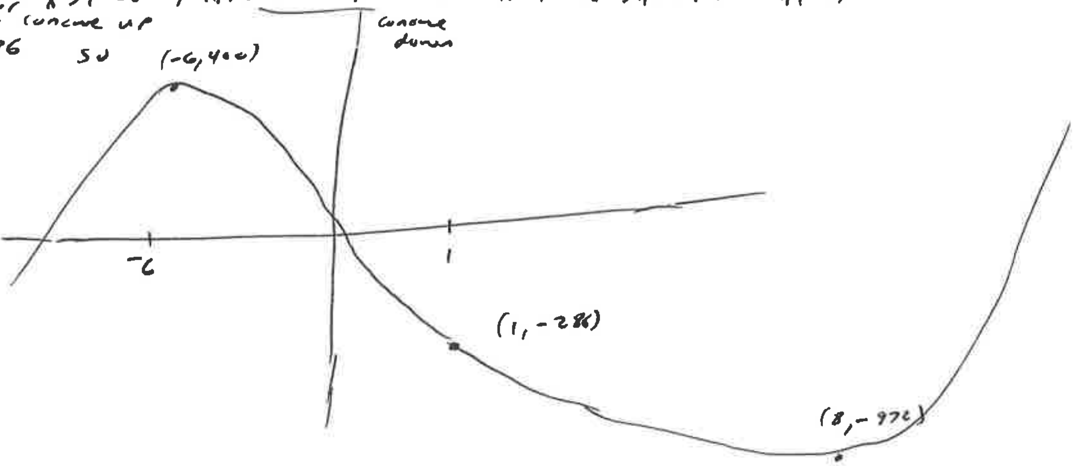
$f'(x) = 0$: $x - 8 = 0$ or $x + 6 = 0 \Rightarrow x = 8, -6$ crit #'s. $f''(8) = 6(8-1) > 0 \Rightarrow x = 8$ rel. min

$f''(-6) = 6(-6-1) < 0$ so rel. max. $f'(0) = -144$ so $f \searrow$ on $(-6, 8)$, $f'(10) = 3(2)(16) > 0$

so $f \nearrow$ on $(8, \infty)$, $f'(-7) = 3(-7-8)(-1) > 0$ so $f \nearrow$ on $(-\infty, -6)$. $f''(x) = 0 \Rightarrow 6(x-1) = 0$

or $x = 1$ but $f''(x) > 0$ for $x > 1$ and $f''(x) < 0$ for $x < 1$ so $x = 1$ is infl. pt. $f(-6) = 400$

$f(8) = -972$, $f(1) = -286$ so



13.) $F(x) = f(g(x))$ by chain rule $F'(x) = f'(g(x)) \cdot g'(x)$ so $F'(5) = f'(g(5)) \cdot g'(5) = f'(-2) \cdot 6$

$= 4 \cdot 6 = 24$