MA 1613 Practice Final Exam Solutions

1.) $f(x)=\frac{x^2-4}{x-1}$, notice $1\notin dom(f)$ & $\lim_{x\to1^-} f(x)=\infty$, $\lim_{x\to1^+} f(x)=-\infty$ so vert asym. at $x=1$

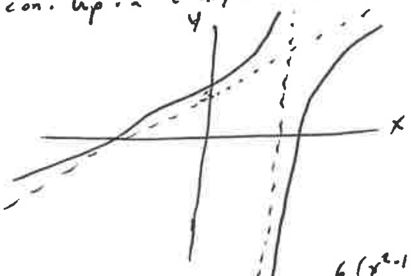
then $f'(x)=\frac{2x(x-1)-(x^2-4)}{(x-1)^2}=\frac{2x^2-2x-x^2+4}{(x-1)^2}=\frac{x^2-2x+4}{(x-1)^2}$. $f''(x)=\frac{(2x-2)(x-1)^2-2(x-1)(x^2-2x+4)}{(x-1)^4}=\frac{-6}{(x-1)^3}$

$f'(x)=0:\Rightarrow x$ no real roots. $f'(x)$ DNE $\Rightarrow x=1$ so $x=1$ crit. #. ~~and~~ vert. asym type $f'(0)=\frac{4}{(-1)^2}>0\Rightarrow f'>0$ for $x<1$

$f'(2)=\frac{4-4+4}{1^2}>0\Rightarrow f'>0$ for $x>1$. $f''>0$ for $x<1$ & $f''<0$ for $x>1$. so $f\nearrow$ on $(-\infty,1)\cup(1,\infty)$

f is con. up on $(-\infty,1)$ & concave dn on $(1,\infty)$. Finally $\frac{x^2-4}{x-1}=x+1-\frac{3}{x-1}$ by long division so has slant asym.

so



2.) $f(x)=\frac{6}{x^2-1}=\frac{6}{(x-1)(x+1)}=6(x^2-1)^{-1}$ notice $\pm1\notin dom(f)$. $\lim_{x\to1^-}f(x)=-\infty$, $\lim_{x\to1^+}f(x)=\infty$, $\lim_{x\to-1^-}f(x)=\infty$, $\lim_{x\to-1^+}f(x)=-\infty$

so vert. asym. at $x=1$ & $x=-1$. $f'(x)=-6(x^2-1)^{-2}(2x)=\frac{-12x}{(x^2-1)^2}$ & $f''(x)=\frac{-12(x^2-1)^2-2(x^2-1)(2x)(-12x)}{(x^2-1)^4}=\frac{12(3x^2+1)}{(x^2-1)^3}$

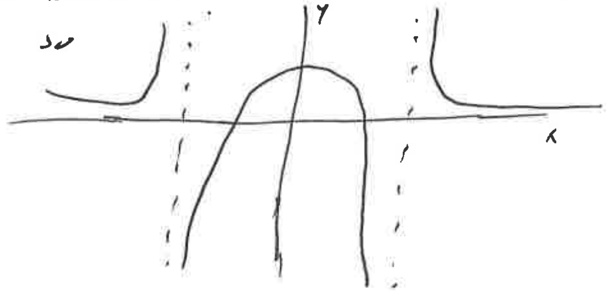
$f'(x)=0$: $-12x=0\Rightarrow x=0$. & $f(0)=-6$ so crit. pt $(0,-6)$. $f'(x)$ DNE $\Rightarrow x=\pm1$ so same as vert. asym.

$f''(0)=-12<0$ so $(0,6)$ local max. $f'(-2)>0$ so $f'>0$ on $(-\infty,-1)$, $f'(-\frac{1}{2})>0$ so $f'>0$ on $(-1,0)$

$f'(\frac{1}{2})<0$ so $f'<0$ on $(0,1)$, $f'(2)<0$ so $f'<0$ on $(1,\infty)$. so $f\nearrow$ on $(-\infty,-1)\cup(-1,0)$ & $f\searrow$ on $(0,1)\cup(1,\infty)$

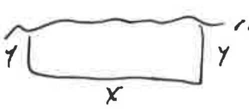
$f''>0$ for $x>1$ & $f''<0$ for $-1<x<1$, $f''>0$ for $x<-1$ so f concave up on $(-\infty,-1)\cup(1,\infty)$

& concave dn on $(-1,1)$. so



$\lim_{x\to\pm\infty} f(x)=0$ so hor. asym

3.)



$P=x+2y$ so $x+2y=240\Rightarrow x=240-2y\Rightarrow A=(240-2y)y\Rightarrow A=240y-2y^2$

$A=xy$ so $A'=240-4y$, then $A'=0\Rightarrow 4y=240$ or $y=60$

$A''=-4\Rightarrow A''(60)<0$ so max. $\therefore x=240-2(60)\Rightarrow x=120$

So $A=(60)(120)=7{,}200$ yd^2

4.) let $x-y=4$ & $P=xy\Rightarrow x=y+4$ so $P=(y+4)y=y^2+4y$ so $P'=2y+4$. $P'=0\Rightarrow 2y+4=0$

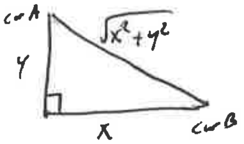
so $2y=-4\Rightarrow y=-2$, $P''=2$ so $P''(-2)>0\Rightarrow$ min. then $x=-2+4=2$ so #s 2 & -2.

5.) $\frac{d}{dx}(x^4y^3-x^6y^9)=\frac{d}{dx}(20)\Rightarrow 4x^3y^3+x^4\cdot3y^2\frac{dy}{dx}-6x^5y^9-x^6\cdot9y^8\frac{dy}{dx}=0$

$\Rightarrow (3x^4y^2-x^6 9y^8)\frac{dy}{dx}=6x^5y^9-4x^3y^3\Rightarrow \frac{dy}{dx}=\frac{6x^5y^9-4x^3y^3}{3x^4y^2-9x^6y^8}$

6.) $\frac{d}{dx}(3x^2y^8+4x^6y^3)=\frac{d}{dx}(29)\Rightarrow 6xy^8+3x^2\cdot8y^7y'+24x^5y^3+4x^6\cdot3y^2y'=0$

$\Rightarrow (24x^2y^7+12x^6y^2)y'=-6xy^8-24x^5y^3\Rightarrow y'=\frac{-6xy^8-24x^5y^3}{24x^2y^7+12x^6y^2}$

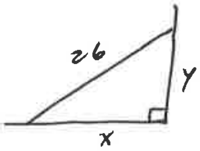
7.) 

let $D=\sqrt{x^2+y^2}$ then $D^2=x^2+y^2$ so $\frac{d}{dt}(D^2)=\frac{d}{dt}(x^2+y^2) \Rightarrow 2DD' = 2xx'+2yy'$

$\Rightarrow DD' = xx'+yy'$. $x(1)=25$, $y(1)=60$ so $D(1)=\sqrt{25^2+60^2}=65$

OTH $x'(t)=25$ & $y'(t)=60$ so $D(1)D'(1) = x(1)x'(1)+y(1)y'(1)$

$\Rightarrow 65D'(1) = 25\cdot 25+60\cdot 60 \Rightarrow D'(1) = \frac{25^2+60^2}{65} = 65$ m/h.

8.) 

so $x^2+y^2=26^2 \Rightarrow 2xx'+2yy'=0 \Rightarrow x(t)x'(t)+y(t)y'(t)=0$

when $x=10 \Rightarrow 10^2+y^2=26^2 \Rightarrow y^2=576 \Rightarrow y=24$ so $10\cdot 5+24y'=0$

$\Rightarrow y'=\frac{-24}{50}=-\frac{12}{25}$ ft/sec.

9.) $y=xe^x-x^2e^{-6x}+x^6$ so, $y'=e^x+xe^x-2xe^{-6x}-x^2e^{-6x}(-6x)'+6x^5 = e^x(1+x)-2xe^{-6x}+6x^2e^{-6x}+6x^5$

10.) $y=5xe^{x^2}+xe^{-9x}+4x^2$, $y'=5e^{x^2}+5xe^{x^2}(x^2)'+e^{-9x}+xe^{-9x}(-9x)'+8x = 5e^{x^2}+10x^2e^{x^2}+e^{-9x}-9xe^{-9x}+8x$

11.) $f(x)=\ln\left(\frac{6x^3-e^x}{x^3+2}\right)$ so $f(x)=\ln(6x^3-e^x)-\ln(x^3+2)$, then $f'(x)=\frac{1}{6x^3-e^x}(6x^3-e^x)'-\frac{1}{x^3+2}(x^3+2)'$

$=\frac{1}{6x^3-e^x}(18x^2-e^x) - \frac{1}{x^3+2}(3x^2) = \frac{18x^2-e^x}{6x^3-e^x}-\frac{3x^2}{x^3+2}$

12.) $f(x)=\frac{e^{3x}\ln(x^2+2)}{x+4}$, $f'(x)=\frac{(e^{3x}\ln(x^2+2))'(x+4)-e^{3x}\ln(x^2+2)}{(x+4)^2} = \frac{\left(e^{3x}(3x)'\ln(x^2+2)+e^{3x}\frac{1}{x^2+2}(x^2+2)'\right)(x+4)-e^{3x}\ln(x^2+2)}{(x+4)^2}$

$=\frac{\left(3e^{3x}\ln(x^2+2)+\frac{2xe^{3x}}{x^2+2}\right)(x+4)-e^{3x}\ln(x^2+2)}{(x+4)^2}$

13.) recall $|x|=\begin{cases}x & x\ge 0\\ -x & x<0\end{cases}$. so for $x>0$ $f(x)=\ln x$ so $f'(x)=\frac{1}{x}$. for $x<0$ $f(x)=\ln(-x)$

so $f'(x)=\frac{1}{-x}(-x)'=\frac{-1}{-x}=\frac{1}{x}$ $\therefore$ $\frac{d}{dx}(\ln|x|)=\frac{1}{x}$.