MA 1723 Practice Exam 1 Solutions

1.) By FTOC, $g'(x) = \cos(x^2)$

2.) By FTOC, $h'(x) = \frac{\sin(x^8+1)}{x^4+\ln(x^2)}(x^2)' - \frac{\sin(x^4+1)}{x^2+\ln x} = \frac{x\sin(x^8+1)}{x^4+\ln(x^2)} - \frac{\sin(x^4+1)}{x^2+\ln x}$

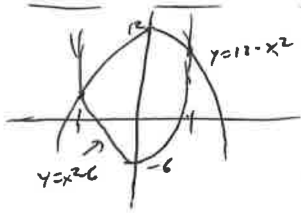
3.) Let $u=\sqrt{x}$, then $du = \frac{1}{2\sqrt{x}}dx$ $\therefore$ $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}dx = 2\int \sin u\,du = -2\cos u + C = -2\cos(\sqrt{x}) + C$

4.) Let $u = x-1$ so $du = dx$ and when $x=1 \Rightarrow u=0$
$x = u+1$ $\qquad x=2 \Rightarrow u=1$

$\therefore \int_1^2 x\sqrt{x-1}\,dx = \int_0^1 (u+1)\sqrt{u}\,du = \int_0^1 (u^{\frac32}+u^{\frac12})\,du = \left(\frac25 u^{\frac52} + \frac23 u^{\frac32}\right)\Big|_0^1$

$= \frac25 + \frac23 = \frac{16}{15}$

5.) $y = 12-x^2$, $y = x^2-6$ have

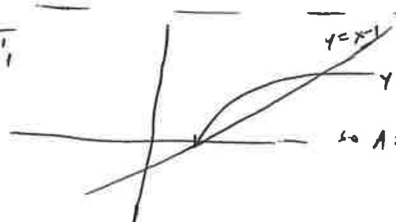


limits are: $12-x^2 = x^2-6 \Rightarrow 18 = 2x^2 \Rightarrow x^2 = 9$ so $x = \pm 3$

so $A = \int_{-3}^{3}\left(12-x^2-(x^2-6)\right)dx = \int_{-3}^{3}(18-2x^2)\,dx = 2\int_0^3 (18-2x^2)\,dx$

$= \left(18x - \frac{2x^3}{3}\right)\Big|_0^3 = (18)(3) - \frac23(3)^3 = 54$

6.) $y = \sqrt{x-1}$ and $x-y=1 \Rightarrow \begin{cases} y = \sqrt{x-1} \\ y = x-1 \end{cases}$



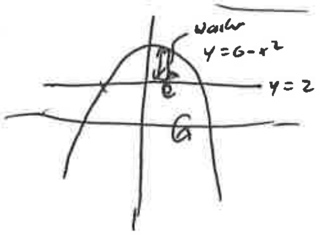
limits: $x-1 = \sqrt{x-1} \Rightarrow (x-1)^2 = x-1 \Rightarrow x^2-2x+1 = x-1$
$\Rightarrow x^2-3x+2=0 \Rightarrow (x-2)(x-1)=0$ so $x=1,2$

so $A = \int_1^2\left(\sqrt{x-1}-(x-1)\right)dx = \int_1^2 \sqrt{x-1}\,dx - \int_1^2 (x-1)\,dx$

$u = x-1$, $du = dx$, $x=1 \Rightarrow u=0$, $x=2 \Rightarrow u=1$

so $A = \int_0^1 u\,du - \left(\frac{x^2}{2}-x\right)\Big|_1^2 = \frac{u^2}{2}\Big|_0^1 - \left(\left[\frac42-2\right]-\left[\frac12-1\right]\right) = \frac12 - \frac12 + 1 = 1$

7.) $y = 6-x^2$, $y=2$
about x-axis

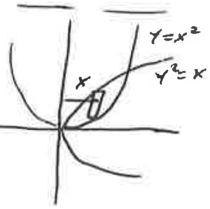


limits: $6-x^2 = 2 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$, washer $= (6-x^2)^2 - 2^2$

so $Vol = \pi\int_{-2}^{2}\left((6-x^2)^2-2^2\right)dx = 2\pi\int_0^2\left((6-x^2)^2-4\right)dx = 2\pi\int_0^2 (36-12x^2+x^4)\,dx$

$= 2\pi\left(36x - 4x^3 + \frac{x^5}{5}\right)\Big|_0^2 = 2\pi\left(72-32+\frac{32}{5}\right) = \frac{4\pi}{5}$

8.) $y=x^2$, $y^2=x$,
about y-axis

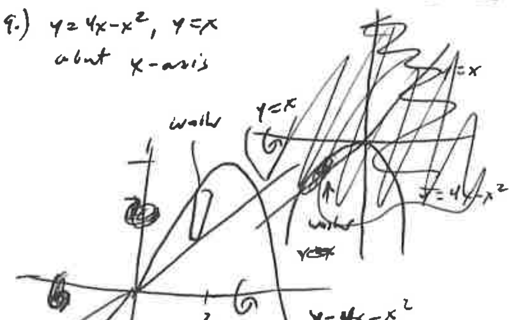


limits $x^2 = \sqrt{x} \Rightarrow x^4 = x \Rightarrow x(x^3-1)=0$ only real roots $x=0, x=1$

so vol $= 2\pi\int_0^1 x(\sqrt{x}-x^2)\,dx = 2\pi\int_0^1\left(x^{\frac32}-x^{\frac52}\right)dx = 2\pi\left(\frac25 x^{\frac52} - \frac27 x^{\frac72}\right)\Big|_0^1$

$= 4\pi\left(\frac15-\frac17\right) = \frac{6\pi}{35}$

9.) $y = 4x-x^2$, $y=x$
about x-axis



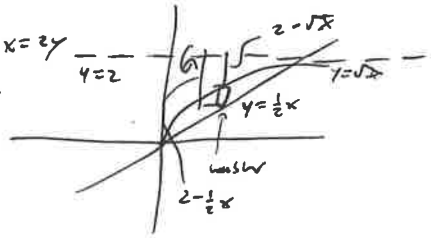
limits: $x = 4x-x^2 \Rightarrow x^2-3x=0 \Rightarrow x(x-3)=0$ $x=0,3$, washer $= (4x-x^2)^2 - x^2$

so $Vol = \pi\int_0^3\left((4x-x^2)^2-x^2\right)dx = \pi\int_0^3 (16x^2-8x^3+x^4-x^2)\,dx$

$= \pi\int_0^3 (15x^2-8x^3+x^4)\,dx = \pi\left(5x^3-2x^4+\frac{x^5}{5}\right)\Big|_0^3$

$= \pi\left(5(27)-2(81)+\frac{243}{5}\right) = \frac{108\pi}{5}$

10.) $y=\sqrt{x}$, $x=2y$
about $y=2$



limits: $\sqrt{x} = \frac{x}{2} \Rightarrow 4x = x^2 \Rightarrow x=0,4$. washer $= (2-\frac{1}{2}x)^2 - (2-\sqrt{x})^2$
$= 4\sqrt{x} - 3x - \frac{1}{4}x^2$

so vol $= \pi \int_0^4 (4x^{\frac{1}{2}} - 3x - \frac{1}{4}x^2)\,dx = \pi \left(\frac{8}{3}x^{\frac{3}{2}} - \frac{3}{2}x^2 - \frac{1}{12}x^3\right)\Big|_0^4$

$= \pi\left(\frac{8}{3}(8) - \frac{3}{2}(16) - \frac{1}{12}(64)\right) = 8\pi$

11.) Hooke's law says $F(x) = kx$ so $W = \int_a^b F(x)\,dx$ let x_0 - natural length of spring so $12 = \int_{x_0}^{x_0+1} kx\,dx = \int_0^1 kx\,dx = k\frac{x^2}{2}\Big|_0^1 = \frac{k}{2}$

$\Rightarrow k = 24$. now $W = \int_0^{\frac{9}{12}} 24x\,dx = 24\frac{x^2}{2}\Big|_0^{\frac{9}{12}} = 12\left(\frac{3}{4}\right)^2 = 12\left(\frac{9}{16}\right) = \frac{27}{4}$

12.) by Hooke's law $40 = k(.15 - .10) = k(.05) \Rightarrow k = 800$. then $W = \int_{.15}^{.18} 800x\,dx = 400x^2\Big|_{.15}^{.18} = 400\left((.18)^2 - (.15)^2\right) = 400(.0099)$
$= \frac{198}{5}$

13.) 1st notice $x \le 1$ so $x^2 \le 1$ $\therefore$ $1 + x^2 \le 2$ $\Rightarrow \sqrt{1+x^2} \le \sqrt{2}$. OTOH $x^2 \ge 0 \Rightarrow x^2+1 \ge 1 \Rightarrow \sqrt{x^2+1} \ge 1$

$\Rightarrow \int_{-1}^1 1\,dx \le \int_{-1}^1 \sqrt{x^2+1}\,dx \le \int_{-1}^1 \sqrt{2}\,dx \Rightarrow 2 \le \int_{-1}^1 \sqrt{x^2+1}\,dx \le 2\sqrt{2}$