

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let $f(x) = x^3 + 3 \sin x + 2 \cos x$. Compute $(f^{-1})'(2)$.

2. Suppose f^{-1} is the inverse function of a differentiable function f and let $g(x) = \frac{1}{f^{-1}(x)}$. If $f(3) = 2$ and $f'(3) = \frac{1}{9}$, compute $g'(2)$.

3. Use logarithmic differentiation to compute the derivative of $y = x^{\sin x}$.

4. Use logarithmic differentiation to compute the derivative of $y = (\sin x)^{\ln x}$.

5. Evaluate the following integral:

$$\int \frac{1+x}{1+x^2} dx .$$

6. Evaluate the following integral:

$$\int \frac{dx}{\sin^{-1}(x)\sqrt{1-x^2}} .$$

7. Evaluate the following limit:

$$\lim_{x \rightarrow 0^+} x^{\sqrt{x}} .$$

8. Evaluate the following limit:

$$\lim_{x \rightarrow \infty} x^{1/x} .$$

9. Evaluate the following integral:

$$\int e^{\sqrt{x}} dx .$$

10. Evaluate the following integral:

$$\int x \ln(x + 1) dx .$$

11. Evaluate the following integral:

$$\int \sin^3 \theta \cos^4 \theta d\theta .$$

12. Evaluate the following integral:

$$\int_0^{\pi/2} \sin^2 x \cos^2 x dx .$$

13. Suppose f is a differentiable function with continuous derivative on $[a, b]$. Show that

$$\int f(x) dx = xf(x) - \int xf'(x) dx .$$

In addition if f has an inverse with $g = f^{-1}$, show that

$$\int_a^b f(x) dx = bf(b) - af(a) - \int_{f(a)}^{f(b)} g(y) dy .$$