

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Evaluate the following integral:

$$\int \frac{\sqrt{x^2 - 1}}{x^4} dx .$$

2. Evaluate the following integral:

$$\int \frac{x^2}{\sqrt{9 - x^2}} dx .$$

3. Evaluate the following integral:

$$\int_0^1 \frac{2}{2x^2 + 3x + 1} dx .$$

4. Evaluate the following integral:

$$\int_{-1}^0 \frac{x^3 - 4x + 1}{x^2 - 3x + 2} dx .$$

5. Determine if the following improper integral converges or diverges. If it converges, evaluate it.

$$\int_0^{\infty} e^{-\sqrt{x}} dx .$$

6. Determine if the following improper integral converges or diverges. If it converges, evaluate it.

$$\int_e^{\infty} \frac{1}{x(\ln x)^2} dx .$$

7. Determine if the following improper integral converges or diverges. If it converges, evaluate it.

$$\int_0^1 \frac{dx}{\sqrt{1 - x^2}} .$$

8. Determine if the following improper integral converges or diverges. If it converges, evaluate it.

$$\int_0^5 \frac{x}{x - 2} dx .$$

9. Find the exact length of the following curve $y = \ln(\cos x)$, $0 \leq x \leq \pi/3$.

10. Find the exact length of the following curve $y = \sqrt{x - x^2} + \sin^{-1}(\sqrt{x})$.

11. Find the exact area of the surface obtained by rotating $y^2 = x + 1$ and $0 \leq x \leq 3$ about the x -axis.

12. Find the exact area of the surface obtained by rotating $y = \sqrt{1 + e^x}$ and $0 \leq x \leq 1$ about the x -axis.

13. Define the following region:

$$\mathcal{R} = \left\{ (x, y) : 1 \leq y \leq \frac{1}{x} \right\} .$$

Show that the solid obtained by rotating \mathcal{R} about the x -axis has finite volume and that it has infinite surface area.