

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

For problems 1 and 2, find the radius of convergence and interval of convergence of the following series:

1.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n5^n} x^n$$

2.
$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n} (x+6)^n$$

For problems 3 and 4, evaluate the following indefinite integral as a power series:

3. $\int \tan^{-1} x \, dx$

4. $\int \frac{dx}{1+x^{11}}$

For problems 5 and 6, using the definition of Taylor series, compute the Taylor series of $f(x)$ at the given point a .

5. $f(x) = \sqrt{x}$ at $a = 16$.

6. $f(x) = e^{-2x}$ at $a = 1$.

For problems 7 and 8, find the points on the curve where the tangent is horizontal or vertical.

7. $x = t^3 - 3t$ and $y = t^3 - 3$

8. $x = \cos \theta$ and $y = \cos(3\theta)$

For problems 9 and 10, find the area of the region that lies inside both curves.

9. $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$

10. $r = a \sin \theta$ and $r = b \cos \theta$ where $a, b > 0$

For problems 11 and 12, find the exact length of the polar curve.

11. $r = 2 \sin \theta$ for $0 \leq \theta \leq 2\pi$

12. $r = 2(1 + \cos \theta)$

13. Show that if $\lim_{n \rightarrow \infty} \sqrt[n]{c_n} = c$ where $c \neq 0$, then the radius of convergence of the power series

$$\sum_{n=1}^{\infty} c_n x^n$$

is $R = \frac{1}{c}$.