Name:______________________________

MA 2733 Section 01 | *Practice Final Exam* | *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let $\mathbf{a}$ and $\mathbf{b}$ be nonzero vectors. Under what conditions is $\text{comp}_{\mathbf{a}}\mathbf{b} = \text{comp}_{\mathbf{b}}\mathbf{a}$?

2. Let $\mathbf{a}$ and $\mathbf{b}$ be nonzero vectors. Under what conditions is $\text{proj}_{\mathbf{a}}\mathbf{b} = \text{proj}_{\mathbf{b}}\mathbf{a}$?

3. Find an equation of the plane through the points $(3, 0, -1)$, $(-2, -2, 3)$, and $(7, 1, -4)$.

4. Find an equation of the plane that passes through the point $(3, 5, -1)$ and contains the line $x = 4 - t$, $y = 2t - 1$, and $z = -3t$.

5. Reduce the following equation to one of the standard forms, classify the surface, and sketch it:

$$x^2 + y^2 - 2x - 6y - z + 10 = 0$$

6. Reduce the following equation to one of the standard forms, classify the surface, and sketch it:

$$x^2 - y^2 + z^2 - 4x - 2z = 0$$

For problems 7 and 8, compute the following limits.

7. $\displaystyle\lim_{t\to 1}\left(\frac{t^2-t}{t-1}\mathbf{i} + \sqrt{t+8}\,\mathbf{j} + \frac{\sin(\pi t)}{\ln t}\mathbf{k}\right)$

8. $\displaystyle\lim_{t\to\infty}\left\langle te^{-t}, \frac{t^3+t}{2t^3-1}, t\sin\left(\frac{1}{t}\right)\right\rangle$

For problems 9 and 10, compute the unit tangent vector $\mathbf{T}(t)$ of $r(t)$ and the indicated point.

9. $r(t) = \langle t^2 - 2t, 1 + 3t, \sin(2t) \rangle$ at the point $t = 2$.

10. $r(t) = \langle \sin^2 t, \cos^2 t, \tan^2 t \rangle$ at the point $t = \dfrac{\pi}{4}$.

For problems 11 and 12, find the unit tangent vector, unit normal vector and the binormal vector; that is $\mathbf{T}(t)$, $\mathbf{N}(t)$, and $\mathbf{B}(t)$ of the vector $\mathbf{r}(t)$.

11. $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$.

12. $\mathbf{r}(t) = \left\langle t^2, \frac{2}{3}t^3, t \right\rangle$ at the point $\left(1, \frac{2}{3}, 1\right)$.

13. Find the curvature of the twisted cube $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$.