

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

For problems 1 and 2, find the radius of convergence and interval of convergence of the following series:

1.  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n5^n} x^n$
2.  $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n} (x+6)^n$

For problems 3 and 4, evaluate the following indefinite integral as a power series:

3.  $\int \tan^{-1} x \, dx$
4.  $\int \frac{dx}{1+x^{11}}$

For problems 5 and 6, using the definition of Taylor series, compute the Taylor series of  $f(x)$  at the given point  $a$ .

5.  $f(x) = \sqrt{x}$  at  $a = 16$ .
6.  $f(x) = e^{-2x}$  at  $a = 1$ .

For problems 7 and 8, find the points on the curve where the tangent is horizontal or vertical.

7.  $x = t^3 - 3t$  and  $y = t^3 - 3$
8.  $x = \cos \theta$  and  $y = \cos(3\theta)$

For problems 9 and 10, find the area of the region that lies inside both curves.

9.  $r = 1 + \cos \theta$  and  $r = 1 - \cos \theta$

10.  $r = a \sin \theta$  and  $r = b \cos \theta$  where  $a, b > 0$

For problems 11 and 12, find the exact length of the polar curve.

11.  $r = 2 \sin \theta$  for  $0 \leq \theta \leq 2\pi$

12.  $r = 2(1 + \cos \theta)$

13. Show that if  $\lim_{n \rightarrow \infty} \sqrt[n]{c_n} = c$  where  $c \neq 0$ , then the radius of convergence of the power series

$$\sum_{n=1}^{\infty} c_n x^n$$

is  $R = \frac{1}{c}$ .