Name:______________________________

MA 3053 Section 01 *Practice Exam 1* *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let $f : X \to Y$ and $g : Y \to Z$. Prove that if $g \circ f$ is an injection, then f is an injection.

2. Let $f : X \to Y$. Given functions $g, h : W \to X$ such that whenever $f \circ g = f \circ h$, then $g = h$; show that f is injective.

3. Let $f : X \to Y$ and $P_\alpha \subseteq Y$ for every $\alpha \in A$ Show

$$f^{-1}\left(\bigcup_{\alpha \in A} P_\alpha\right) = \bigcup_{\alpha \in A} f^{-1}(P_\alpha)$$

4. Let $f : X \to Y$ and $P_\alpha \subseteq X$ for every $\alpha \in A$ Show

$$f\left(\bigcup_{\alpha \in A} P_\alpha\right) = \bigcup_{\alpha \in A} f(P_\alpha)$$

5. Let $\sim$ be a relation on $X = \mathbb{Z} \times \mathbb{Z}$ by $(a, b) \sim (c, d)$ if and only if $a + d = b + c$. Show $\sim$ is an equivalence relation on X.

6. Let $f : X \to Y$. Let $\sim$ be a relation on X by $x \sim y$ if and only if $f(x) = f(y)$. Show $\sim$ is an equivalence relation on X.

7. Let $\mathcal{F}$ be a family of sets and let $\preceq$ be a relation on $\mathcal{F}$ by $X \preceq Y$ if and only if $X \subset Y$. Show $\preceq$ is a partial order on $\mathcal{F}$.

8. Let $\preceq$ be a relation on $\mathbb{R}^n$ defined as follows: Let $x = (a_1, \ldots, a_n)$ and $y = (b_1, \ldots, b_n)$ be distinct elements of $\mathbb{R}^n$. Let $k \in \mathbb{N}^+$ be the least number such that $a_k \neq b_k$, then define $x \preceq y$ if and only if $a_k < b_k$. Show $\preceq$ is a partial order on $\mathbb{R}^n$.

9. What are the multiplication and addition tables for the congruence classes in $\mathbb{Z}/14\mathbb{Z}$.