

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let P and Q be statements. Prove the deMorgans laws:

(a) $\neg(P \wedge Q) = \neg P \vee \neg Q$

(b) $\neg(P \vee Q) = \neg P \wedge \neg Q$

2. Let P and Q be statements. Prove that $(P \implies Q) \iff (\neg P \vee Q)$.

3. Prove that $\sqrt{10}$ is irrational.

4. Prove there exists irrational numbers x and y such that x^y is rational.

5. Prove that for any $n \in \mathbb{N}$ the following holds:

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

6. Let $n \in \mathbb{N}$. Prove that

$$\sum_{k=0}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

7. Prove that for any $n \in \mathbb{N}$, $2^n > n$ holds.

8. Assume that there is a polynomial, $p(n)$, of degree 3 such that

$$p(n) = \sum_{k=0}^n k^2$$

Find the formula for $p(n)$ and prove that the formula is correct.

9. Let P and Q be statements. Prove that the following statement is always true:

$$[P \wedge (P \Rightarrow Q)] \Rightarrow Q$$