Name:______________________________

MA 3053 Section 01 | *Practice Exam 1* | *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let $f : X \to Y$ and $g : Y \to Z$. Prove that if $g \circ f$ is an injection, then f is an injection.

2. Let $f : X \to Y$. Given functions $g, h : W \to X$ such that whenever $f \circ g = f \circ h$, then $g = h$; show that f is injective.

3. Let $f : X \to Y$ and $P_\alpha \subseteq Y$ for every $\alpha \in A$ Show

$$f^{-1}\left(\bigcap_{\alpha \in A} P_\alpha\right) = \bigcap_{\alpha \in A} f^{-1}(P_\alpha)$$

4. Let $f : X \to Y$ and $P_\alpha \subseteq X$ for every $\alpha \in A$ Show

$$f\left(\bigcup_{\alpha \in A} P_\alpha\right) = \bigcup_{\alpha \in A} f(P_\alpha)$$

5. Let $\sim$ be a relation on $X = \mathbb{Z} \times \mathbb{Z}$ by $(a, b) \sim (c, d)$ if and only if $a + d = b + c$. Show $\sim$ is an equivalence relation on X.

6. Let $\sim$ be a relation on $X = \mathbb{Z} \times \mathbb{N}^+$ by $(a, b) \sim (c, d)$ if and only if $ad = bc$. Show $\sim$ is an equivalence relation on X.

7. Let $\mathcal{F}$ be a family of sets and let $\preceq$ be a relation on $\mathcal{F}$ by $X \preceq Y$ if and only if $X \subseteq Y$. Show $\preceq$ is a partial order on $\mathcal{F}$.

8. Let X be a set and $P = \{f : f : X \to X\}$. Define the relation $\preceq$ on P by $f \preceq g$ if and only if $f(x) \leq g(x)$ for all $x \in X$. Prove $\preceq$ is a partial order on P.

9. What are the multiplication and addition tables for the congruence classes in $\mathbb{Z}/6\mathbb{Z}$.