Name:______________________________

MA 3053 Section 01 — *Practice Exam 1* — *January 30, 2020*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let $f : X \to X$. Suppose f has the property that $f \circ f = \mathrm{id}|_X$, that is $(f \circ f)(x) = x$ for all $x \in X$. Prove that f is a bijection.

2. Let A and B be nonempty sets and let $f : A \to C$, $g : B \to D$ be functions. Define $h : A \times B \to C \times D$ via $h(a, b) = (f(a), g(b))$. Prove or disprove that if h is an injection, then f and g must also be injections.

3. Let A, B, C, and D be sets. Prove that

$$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D) .$$

4. Let $f : X \to Y$ be an injection and $P_\alpha \subseteq X$ for every $\alpha \in A$. Show that

$$f\left(\bigcap_{\alpha \in A} P_\alpha\right) = \bigcap_{\alpha \in A} f(P_\alpha)$$

5. Let $\preceq$ be the relation on $\mathbb{N}^+$ defined by $x \preceq y$ if and only if there is a $z \in \mathbb{N}^+$ such that

$$xz = y .$$

Prove that $\preceq$ is a partial ordering on $\mathbb{N}^+$.

6. Let X be a set and $P = \{f : f : X \to X\}$. Define the relation $\preceq$ on P by $f \preceq g$ if and only if $f(x) \leq g(x)$ for all $x \in X$. Prove $\preceq$ is a partial order on P.

7. Let $f : X \to X$ be a function. Define the relation $\sim$ on X by $x \sim y$ if and only if $f(x) = f(y)$. Prove that $\sim$ is an equivalence relation on X. What are the equivalence classes in the quotient space $X/\sim$, be sure to justify.

8. Let $\sim$ be a relation on $X = \mathbb{Z} \times \mathbb{Z}$ by $(a, b) \sim (c, d)$ if and only if $a + d = b + c$. Show $\sim$ is an equivalence relation on X.

9. Let A, B, and C be sets. Prove or disprove that if $A \cup B \neq A \cap C$, then $A \nsubseteq C$ or $B \nsubseteq A$.