Name:______________________________

Math 3053 Section 01 — *Practice Exam 2* — *February 24, 2020*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let P and Q be statements. Prove that the following statement is always true:

$$[P \wedge (P \Rightarrow Q)] \Rightarrow Q\,.$$

2. Let P and Q be statements. Prove that $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$.

3. Prove that $\sqrt{14}$ is irrational.

4. Prove there exists irrational numbers x and y such that x^y is rational.

5. Suppose $f : \mathbb{R} \to \mathbb{R}$ is an odd function that is differentiable everywhere. Prove that for every positive number b, there is a number $c \in (-b, b)$ such that $f'(c) = f(b)/b$. (HINT: You will need the Mean Value Theorem)

6. Suppose a,b, and c are all positive real numbers. Prove that if $ab = c$, then either $a \leq \sqrt{c}$ or $b \leq \sqrt{c}$.

7. Let $n \in \mathbb{N}$. Prove that

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}\,.$$

8. Let f be a real function such that for $x, y \in \mathbb{R}$,

$$f(x+y) = f(x) + f(y)\,.$$

Prove that:

(a) $f(0) = 0$

(b) $f(n) = nf(1)$, for any $n \in \mathbb{N}$.

9. Prove the product of n rational numbers is again a rational number. Is the product of two irrational numbers always irrational? Prove or disprove you claim.