

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Determine if the following set of vectors are linear independent or not:

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right\}$$

2. Determine if the following set of vectors are linear independent or not:

$$\left\{ \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 2 \\ -6 \end{pmatrix}, \begin{pmatrix} 9 \\ 4 \\ -8 \end{pmatrix} \right\}$$

3. Let $T : C^4(\mathbb{R}) \rightarrow C(\mathbb{R})$ with

$$T(f) = \frac{d^4 f}{dx^4}$$

Show that T is a linear transformation.

4. Let $T : C(\mathbb{R}) \rightarrow \mathbb{R}$ with

$$T(f) = \sum_{j=1}^n f(x_j)$$

where x_1, \dots, x_n are a set of random real numbers. Show that T is a linear transformation.

5. Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ be a linear transformation defined by:

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2x_1 - x_2 \\ 10x_2 + 2x_3 \\ 4x_2 + 5x_4 \\ 11x_2 - 8x_4 \end{pmatrix}$$

Find the matrix that represents T .

6. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear transformation defined by:

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + 4x_2 + 5x_3 \\ 3x_2 - 2x_3 \end{pmatrix}$$

Find the matrix that represents T .

7. Let

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix}$$

It is known that A is invertible. Compute A^{-1} using row reduction.

8. Let

$$A = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}$$

It is known that A is invertible. Compute A^{-1} using row reduction.

9. Let v_1, \dots, v_k be a set of linear independent vectors in \mathbb{R}^n . Suppose A is an $n \times n$ matrix. Show that Av_1, \dots, Av_k are linear independent.