Name:______________________________

Math 3113 Section 01 | *Practice Exam 3* | *November 5, 2015*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Let $T : P_2 \to P_2$ be a linear transformation defined by

$$T(a_0 + a_1 t + a_2 t^2) = 3a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2$$

Find a matrix A, representing T with respect to the basis $B = \{1, t, t^2\}$ and find the eigenvalues and eigenvectors of T.

2. Let $T : P_1 \to P_1$ be a linear transformation defined by

$$T(a_0 + a_1 t) = (2a_0 + 7a_1) + (7a_0 + 2a_1)t$$

Find a matrix A, representing T with respect to the basis $B = \{1, t\}$ and find the eigenvalues and eigenvectors of T.

3. Compute the eigenvalues and eigenvectors of A if

$$A = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$$

4. Compute the eigenvalues and eigenvectors of A if

$$A = \begin{pmatrix} 3 & 1 \\ -2 & 5 \end{pmatrix}$$

5. Let u be a vector in $\mathbb{R}^n$. Define $T : \mathbb{R}^n \to \mathbb{R}^m$ by

$$T(x) = \text{Proj}_u(x) = \frac{\langle x, u \rangle}{\|u\|^2} u$$

Show that T is a linear transformation. Here, $\langle v_1, v_2 \rangle$ is just the dot product in $\mathbb{R}^n$.

6. Let $W = \text{span}\{v_1, \ldots, v_k\}$ be a subspace of $\mathbb{R}^n$. Define $T : \mathbb{R}^n \to \mathbb{R}^m$ by

$$T(x) = \text{Proj}_W(x) = \sum_{j=1}^{k} \frac{\langle x, v_j \rangle}{\|v_j\|^2} v_j$$

The previous problem will generalize to show that T is a linear transformation. Show this. Moreover compute the kernel of T. (HINT: you won't need to compute any thing to deduce the kernel)

7. Let $W = \text{span}\{v_1, v_2, v_3\}$ be a subspace of $\mathbb{R}^4$ where

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

Using the Gram-Schmidt process, find an orthonormal basis for W.

8. Let $W = \text{span}\{v_1, v_2\}$ be a subspace of $\mathbb{R}^3$ where

$$v_1 = \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}, v_2 = \begin{pmatrix} -3 \\ 14 \\ -7 \end{pmatrix}$$

Using the Gram-Schmidt process, find an orthonormal basis for W.

9. Let P be a $n \times n$ matrix whose columns are orthonormal. Show $\langle Px, Py \rangle = 0$ if and only if $\langle x, y \rangle = 0$. Here $\langle v_1, v_2 \rangle$ is the dot product in $\mathbb{R}^n$