Name:____________________________

MA 3113 Section 51 | *Practice Exam 1* | *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit! On the actual exam the last page will have a list of matrices and their row reduced echelon or row echelon forms that you may or may not need.

1. Determine if the following set of vectors are linear independent or not:

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right\}$$

2. Determine if the following set of vectors are linear independent or not:

$$\left\{ \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 2 \\ -6 \end{pmatrix}, \begin{pmatrix} 9 \\ 4 \\ -8 \end{pmatrix} \right\}$$

3. Let $T : C^1(\mathbb{R}) \to C(\mathbb{R})$ with

$$T(f) = \frac{df}{dx}$$

Show that T is a linear transformation.

4. Let $T : C(\mathbb{R}) \to \mathbb{R}$ with

$$T(f) = \sum_{j=1}^{n} f(x_j)$$

where $x_1, \ldots, x_n$ are a set of random real numbers. Show that T is a linear transformation.

5. Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation defined by:

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2x_1 - x_2 \\ 10x_2 + 2x_3 \\ 4x_2 + 5x_4 \\ 11x_2 - 8x_4 \end{pmatrix}$$

Find the matrix that represents T.

6. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by:

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + 4x_2 + 5x_3 \\ 3x_2 - 2x_3 \end{pmatrix}$$

Find the matrix that represents T.

7. Let

$$A = \begin{pmatrix} 3 & -6 \\ -1 & 2 \end{pmatrix}.$$

Construct a 2×2 nonzero matrix B such that $AB = 0$.

8. Let

$$A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 4 & -5 \\ 3 & k \end{pmatrix}.$$

For what value(s) of k, if any, will yield $AB = BA$?

9. Suppose $(B - C)D = 0$, where B and C are $m \times n$ matrices and D is an $n \times n$ invertible matrix. Show that $B = C$.

10. Let A be an $n \times n$ invertible matrix and $\lambda \in \mathbb{C}$ be arbitrary. Consider the following equation:

$$(A - \lambda I_n)v = 0 \ .$$

Suppose the above equation has a nonzero solution, v, for some fixed λ. Show that $\lambda \neq 0$.

11. Let

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix}$$

It is known that A is invertible. Compute A^{-1} using row reduction.

12. Let

$$A = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}$$

It is known that A is invertible. Compute A^{-1} using row reduction.

13. Let $v_1, \ldots, v_k$ be a set of linear independent vectors in $\mathbb{R}^n$. Suppose A is an $n \times n$ matrix. Is it always true that $Av_1, \ldots, Av_k$ must be linear independent in $\mathbb{R}^n$? Why or why not?