Name:______________________________

MA 3113 Section 51 *Practice Final Exam* *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit! On the actual exam the last page will have a list of matrices and their row reduced echelon or row echelon forms that you may or may not need.

1. Compute the eigenvalues and eigenvectors of A if

$$A = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$$

2. Compute the eigenvalues and eigenvectors of A if

$$A = \begin{pmatrix} 3 & 1 \\ -2 & 5 \end{pmatrix}$$

3. Let P be a square matrix with the property that $P^2 = P$. What are the possible eigenvalues of P? Be sure to justify.

4. Show that if A^2 is the zero matrix, then the only possible eigenvalue is 0.

5. Let λ be an eigenvalue for a square matrix A. Show that λ^k is an eigenvalue for A^k for some natural number k.

6. Suppose a square matrix A has 0 as an eigenvalue. Show that A cannot be invertible.

7. Use the Gram-Schmidt process to find an orthonormal basis for $S = \{1, t, t^2\}$ the standard basis for P_2 with the following inner product

$$\langle f, g \rangle = \int_0^1 f(t)g(t)\ dt\ .$$

8. Use the Gram-Schmidt process to find an orthonormal basis for $S = \{1, t\}$ the standard basis for P_1 with the following inner product

$$\langle f, g \rangle = \int_0^1 f(t)g(t)\ dt\ .$$

9. Let P be a $n \times n$ matrix whose columns are orthonormal. For $\mathbf{x} \in \mathbb{R}^n$, show that $\|P\mathbf{x}\| = \|\mathbf{x}\|$. (HINT: If the columns of P are orthonormal than what is P^tP equal to?)

10. Let P be a $n \times n$ matrix whose columns are orthonormal. Show $\langle Px, Py \rangle = 0$ if and only if $\langle x, y \rangle = 0$. Here $\langle v_1, v_2 \rangle$ is the dot product in $\mathbb{R}^n$

11. Let A be a square matrix with real entries such that $A = A^t$. Show that if λ is an eigenvalue of A, then λ must be real, that is $\lambda = \overline{\lambda}$. (HINT: Consider $\langle Av, v \rangle$, here $\langle u_1, u_2 \rangle$ is just the usual dot product in $\mathbb{R}^n$.)

12. Let A be a square matrix with real entries such that $A = A^t$. Show that if $\lambda_1 \neq \lambda_2$ are eigenvalues, then the corresponding eigenvectors v_1 and v_2 must be orthogonal. (HINT: use the hint from the previous problem.)