Name:____________________________

Math 3163 Section 01 | *Practice Exam 2* | *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let G be an Abelian group of order $|G| = 16$. Suppose there are elements $a, b \in G$ such that $|a| = |b| = 4$ and $a^2 \neq b^2$. Determine the isomorphism class of G.

2. Let p, q, and r be distinct primes and G an Abelian group such that $|G| = pqr$. What are the possible groups G is isomorphic to?

3. Let R be a commutative ring with unity. Let A and B be ideals of R such $A + B = R$. Define

$$AB = \{a_1b_1 + a_2b_2 + \cdots + a_nb_n : a_i \in A,\ b_i \in B, n \geq 0\}$$

Prove that $A \cap B \subset AB$.

4. Define the following ring:

$$\mathbb{Z}[x,y] = \left\{ a = \sum_{k=1}^{n} \sum_{j=1}^{m} a_{kj} x^k y^j \ : \ a_{kj} \in \mathbb{Z} \right\}$$

Prove that $I = \langle 2, x, y \rangle$ is a maximal ideal in $\mathbb{Z}[x,y]$.

5. Let R be a ring with unity and let $a \in R$ be a unit. Define $\varphi : R \to R$ via $\varphi(x) = axa^{-1}$. Prove that φ is a ring homomorphism.

6. Let R be the following ring:

$$R = \left\{ A = \begin{pmatrix} a & b \\ b & a \end{pmatrix} : a, b \in \mathbb{Z} \right\}$$

Define the map $\varphi : R \to \mathbb{Z}$ via $\varphi(A) = a - b$. Show φ is a ring homomorphism. What is the kernel of φ?

7. Construct a field with 27 elements. Be sure to justify why your construction is a field.

8. Suppose that $f(x) \in Z_m[x]$. What criteria is needed on $f(x)$ and m such that $Z_m[x]/\langle f(x) \rangle$ is field with m^n elements. Be sure to justify your conditions.

9. Prove that the ideal $I = \langle 2 + i \rangle$ is a maximal ideal in $\mathbb{Z}[i]$.