Name:____________________________

Math 3163 Section 01 *Practice Final Exam* *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let p be a prime and F be a field of characteristic p. Show that $f(x) = x^{p^n} - x$ over F has distinct zeros.

2. Let p be a prime and $F = Z_p(t)$, the field of quotients of the ring $Z_p[x]$, and let $f(x) = x^p - t$. Prove that $f(x)$ is irreducible over F and has a multiple zero in $K = F[x]/\langle x^p - t\rangle$.

3. Find the minimal polynomial for $\sqrt{-1} + \sqrt{3}$ over $\mathbb{Q}$ and prove that that the minimal polynomial is irreducible over $\mathbb{Q}$.

4. Find the minimal polynomial for $\sqrt[3]{2} + \sqrt[3]{4}$ over $\mathbb{Q}$ and prove that that the minimal polynomial is irreducible over $\mathbb{Q}$.

5. Let a be a complex number that is algebraic over $\mathbb{Q}$. Show that $\sqrt{a}$ is algebraic over $\mathbb{Q}$ and deduce that $\sqrt[2n]{a}$ is algebraic over $\mathbb{Q}$.

6. Let F be a field and let α and β be transcendental numbers over F. Prove that either $\alpha\beta$ or $\alpha + \beta$ is also transcendental over F.

7. Let $p(x) = x^3 - 2$. Show that $p(x) \in \mathbb{Q}[x]$ is irreducible. Compute the splitting field of $p(x)$ over $\mathbb{Q}$ and construct the Galois group of $p(x)$ over $\mathbb{Q}$. Finally construct the lattice diagram for splitting field over $\mathbb{Q}$ and the lattice diagram for the Galois group.

8. Let $p(x) = x^4 - 7x^2 + 10$. Show that $p(x) \in \mathbb{Q}[x]$ is reducible. Compute the splitting field of $p(x)$ over $\mathbb{Q}$ and construct the Galois group of $p(x)$ over $\mathbb{Q}$. Finally construct the lattice diagram for splitting field over $\mathbb{Q}$ and the lattice diagram for the Galois group.

9. Let p be an odd prime. Set $q(x) = x^p - 1$. Show that $q(x) \in \mathbb{Q}[x]$ is reducible. Compute the splitting field of $q(x)$ over $\mathbb{Q}$ and construct the Galois group of $q(x)$ over $\mathbb{Q}$. Finally construct the lattice diagram for splitting field over $\mathbb{Q}$ and the lattice diagram for the Galois group.