Name: ____________________

MA 3163 Section 01 *Practice Exam 1* *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution you will receive little or no credit!

1. Let G be a group and let $g \in G$ have order 2. Show that $K = Z(G) \cup gZ(G)$ is a subgroup of G.

2. Let H be a subgroup of a group G. Define the set

$$N(H) = \{x \in G : x^{-1}Hx = H\}$$

Prove that $N(H)$ is a subgroup of G.

3. Let G be a cyclic group with $|G| = 2 \cdot 5 \cdot 7$. List all the subgroups: That is write down the generator for each subgroup, its order and write down the subgroup lattice diagram.

4. Let G be a cyclic group with $|G| = 3^2 \cdot 5$. List all the subgroups: That is write down the generator for each subgroup, its order and write down the subgroup lattice diagram.

5. Let $G = S_3$ and $H = \{(1), (12)\}$. Compute the left cosets of H in G.

6. Let $G = Z_9$ and $H = \{0, 3, 6\}$. Compute the left cosets of H in G.

7. Let G be a group and let $\varphi : G \to G$ be an automorphism. Define

$$H = \left\{g \in G : \varphi^2(g) = g\right\}$$

Prove H is a subgroup of G.

8. Let G be a group and let $g, h \in G$. Suppose g and h induce the same inner automorphism of G. Prove that $h^{-1}g \in Z(G)$.

9. Let p be a prime and G be a group. Suppose that $|G : Z(G)| = p^2$. Prove that

$$G/Z(G) \cong Z_p \oplus Z_p$$