Name:____________________________

Math 3163 Section 01 *Practice Final Exam* *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Construct a field with 27 elements. Be sure to justify why your construction is a field.

2. Suppose that $f(x) \in Z_m[x]$. What criteria is needed on $f(x)$ and m such that $Z_m[x]/\langle f(x)\rangle$ is field with m^n elements. Be sure to justify your conditions.

3. Show $\mathbb{Q}(4-i) = \mathbb{Q}(1+i)$.

4. Let $a, b \in \mathbb{Q}$ with $a \neq 0$. Show $\mathbb{Q}(\sqrt{a}) = \mathbb{Q}(\sqrt{b})$ if and only if there exists some $c \in \mathbb{Q}$ such that $a = bc^2$.

5. Let F be a field and $p(x) = x^3 + x + 1 \in F[x]$ such that $p(x)$ is irreducible over F. Let a be a zero to $p(x)$ and express a^{-1} in terms of the basis elements for $F(a)$. What does this say for a^{-k} in relation to $F(a)$ for some integer k.

6. Let $f(x)$ be a nonconstant element of $F[x]$. If a belongs to some extension of F and $f(a)$ is algebraic over F, prove that a is algebraic over F.

7. Let $p(x) = x^3 - 2$. Show that $p(x) \in \mathbb{Q}[x]$ is irreducible. Compute the splitting field of $p(x)$ over $\mathbb{Q}$ and construct the Galois group of $p(x)$ over $\mathbb{Q}$. Finally construct the lattice diagram for splitting field over $\mathbb{Q}$ and the lattice diagram for the Galois group.

8. Let $p(x) = x^4 - 7x^2 + 10$. Show that $p(x) \in \mathbb{Q}[x]$ is reducible. Compute the splitting field of $p(x)$ over $\mathbb{Q}$ and construct the Galois group of $p(x)$ over $\mathbb{Q}$. Finally construct the lattice diagram for splitting field over $\mathbb{Q}$ and the lattice diagram for the Galois group.

9. Let p be an odd prime. Set $q(x) = x^p - 1$. Show that $q(x) \in \mathbb{Q}[x]$ is reducible. Compute the splitting field of $q(x)$ over $\mathbb{Q}$ and construct the Galois group of $q(x)$ over $\mathbb{Q}$. Finally construct the lattice diagram for splitting field over $\mathbb{Q}$ and the lattice diagram for the Galois group.