Name:____________________________

Math 4943/6943 Section 01 *Practice Final Exam* *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let S be a surface parametrized by

$$x(u,v) = \left(u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2\right) .$$

Compute the second fundamental form, the Gaussian curvature, the mean curvature and the principle curvatures.

2. Let M be the Möbius strip parametrized by

$$x(u,v) = \left(\left(2 - v\sin\left(\frac{u}{2}\right)\right)\sin u, \left(2 - v\sin\left(\frac{u}{2}\right)\right)\cos u, v\cos\left(\frac{u}{2}\right)\right) .$$

Compute the second fundamental form, the Gaussian curvature, the mean curvature and the principle curvatures.

3. A diffeomorphism $\varphi : S \to \tilde{S}$ is said to be **area-preserving** if the area of any region $R \subseteq S$ is equal to the area of $\varphi(R)$. Prove that if φ is area-preserving and conformal, then φ is an isometry.

4. Let S_1 and S_2 be regular surfaces. Prove that if $\varphi : S_1 \to S_2$ is an isometry, then $\varphi^{-1} : S_2 \to S_1$ is also an isometry.

5. Let S be a surface of revolution parametrized by

$$x(u,v) = (\varphi(v)\cos u, \varphi(v)\sin u, \psi(v))$$

for $0 < u < 2\pi$, $a < v < b$ and $\varphi > 0$. Compute the Christoffel symbols.

6. Compute the Christoffel symbols for an open set of the plane in Cartesian coordinates.

7. Let T^2 be the torus. Describe the Gauss map of T^2 and prove that

$$\int\int_T K \, d\sigma = 0 \, .$$

8. Let Γ_1 and Γ_2 be two simple closed geodesics on a compact connected surface S of positive curvature. Prove that Γ_1 and Γ_2 must intersect.